

SULIT



Second Semester Examination
2017/2018 Academic Session

May / June 2018

MAA101 - Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN (7)** pages of printed materials before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all six (6)** questions.

[Arahan : Jawab **semua enam (6)** soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

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Question 1

- (a) Suppose $f(x) = \sqrt{x}$ and $g(x) = x^3$. Find the following compositions with their domains and ranges:

(i) $f \circ g$

(ii) $g \circ f$

(iii) $f \circ f$

[15 marks]

- (b) Suppose $f(x) = \sqrt{x^3 + 2x + 4}$ is defined on its natural domain. Give the new function if f is shifted 4 units up and compressed horizontally with a factor of 2.

[5 marks]

- (c) Given $f(x) = \frac{x-6}{x+4}$, $x \neq -4$. Is f one-to-one? If it is, find $f^{-1}(x)$ and its domain.

[10 marks]

Soalan 1

- (a) Andaikan $f(x) = \sqrt{x}$ dan $g(x) = x^3$. Cari komposisi berikut beserta domain dan julatnya.

(i) $f \circ g$

(ii) $g \circ f$

(iii) $f \circ f$

[15 markah]

- (b) Andaikan $f(x) = \sqrt{x^3 + 2x + 4}$ ditakrifkan pada domain aslinya. Berikan fungsi baru jika f dianjak 4 unit ke atas dan dimampatkan secara mengufuk dengan faktor 2.

[5 markah]

- (c) Diberi $f(x) = \frac{x-6}{x+4}$, $x \neq -4$. Adakah f satu-ke-satu? Jika ya, cari $f^{-1}(x)$ dan domainnya.

[10 markah]

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SULIT****Question 2**

(a) Compute the following limits:

(i) $\lim_{h \rightarrow 0} \frac{100}{(10h-1)^4+2}$

(ii) $\lim_{t \rightarrow 3} \left(4t - \frac{2}{t-3}\right)(6+t-t^2)$

(iii) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{4x+5}-3}$

[12 marks]

(b) Explain why $\lim_{x \rightarrow 3^+} \sqrt{\frac{x-3}{2-x}}$ does not exist.

[4 marks]

(c) If $0 \leq x^2 \sec^2 x \leq x^4 + x^2$, find $\lim_{x \rightarrow 0} \left(x^2 \sec^2 x + \frac{\tan x}{x}\right)$.
(Hint: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$)

[8 marks]

(d) Determine all vertical and oblique asymptotes for $y = \frac{x^4}{x^3+0}$.

[8 marks]

(e) Use the Intermediate Value Theorem to verify that $f(x) = x^3 - x - 1$ has a zero in the interval $[1,2]$.

[8 marks]

Soalan 2

(a) *Kirakan had-had berikut:*

(i) $had_{h \rightarrow 0} \frac{100}{(10h-1)^4+2}$

(ii) $\lim_{t \rightarrow 3} \left(4t - \frac{2}{t-3}\right) (6 + t - t^2)$

(iii) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{4x+5}-3}$

[12 markah]

(b) Terangkan sebab $\lim_{x \rightarrow 3^+} \sqrt{\frac{x-3}{2-x}}$ tidak wujud.

[4 markah]

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(c) Jika $0 \leq x^2 \sec^2 x \leq x^4 + x^2$, cari $\lim_{x \rightarrow 0} \left(x^2 \sec^2 x + \frac{\tan x}{x}\right)$.
(Petunjuk: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$)

[8 markah]

(d) Tentukan kesemua asimptot menegak dan serong bagi $y = \frac{x^4}{x^3+0}$.

[8 markah]

(e) Gunakan Teorem Nilai Pertengahan untuk mengesahkan bahawa $f(x) = x^3 - x - 1$ mempunyai satu punca dalam selang $[1,2]$.

[8 markah]

Question 3

(a) Use the definition of the derivative to find $\frac{d}{dx}(ax^2 + bx + c)$ where a, b and c are constants.

[10 marks]

(b) Find the derivatives of the following functions:

(i) $f(x) = 4\sqrt{t} - \frac{1}{4}t^4 + t + 1$

(ii) $f(x) = (x+2)\ln(x^2+1)$

(iii) $f(x) = \sin^2(x^3 + 2x)$

(iv) $f(x) = \frac{4x}{x+2}$

[24 marks]

- (c) Consider a curve with equation $x^4 + y^4 = 2$. Use the implicit differentiation to find $\frac{dy}{dx}$. Then, find the slope of the curve at $(1, -1)$.

[6 marks]

Soalan 3

- (a) Gunakan takrif bagi terbitan untuk mencari $\frac{d}{dx}(ax^2 + bx + c)$ yang mana a , b , dan c adalah pemalar.

[10 markah]

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- (b) Cari terbitan bagi fungsi berikut:

(i) $f(x) = 4\sqrt{t} - \frac{1}{4}t^4 + t + 1$

(iii) $f(x) = (x + 2) \ln(x^2 + 1)$

(iv) $f(x) = \sin^2(x^3 + 2x)$

(iv) $f(x) = \frac{4x}{x+2}$

[24 markah]

- (c) Pertimbangkan lengkung dengan persamaan $x^4 + y^4 = 2$. Gunakan pembezaan tersirat untuk mencari $\frac{dy}{dx}$. Seterusnya, cari kecerunan lengkung tersebut pada $(1, -1)$.

[6 markah]

Question 4

- (a) Given $f(x) = (x^2 - 2x - 11)^2$, $f'(x) = 4(x - 1)(x^2 - 2x - 11)$ and $f''(x) = 12(x - 3)(x + 1)$, find
- (i) all the x - and y -intercepts and the asymptotes,

- (ii) the intervals on which f is increasing or decreasing,
- (iii) the local maximum and minimum points, if any, and
- (iv) the intervals of concavity and the inflection points, if exist.
Then, sketch the graph of f .
- [30 marks]
- (b) A three-sided fence is to be built next to a straight section of a river, which forms the fourth side of a rectangular region. There is 96 meters of fencing available. Find the maximum enclosed area and the dimensions of the rectangular region.
- [10 marks]

Soalan 4

- (a) Diberi $f(x) = (x^2 - 2x - 11)^2$, $f'(x) = 4(x - 1)(x^2 - 2x - 11)$ dan $f''(x) = 12(x - 3)(x + 1)$, dapatkan
- (i) kesemua pintasan- x dan - y dan asimptot,
- (ii) selang bagi f menokok atau menyusut,

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- (iii) titik maksimum dan titik minimum tempatan, jika ada, dan
- (iv) selang kecekungan dan titik lengkung balas, jika wujud.
Seterusnya, lakarkan graf f .
- [30 markah]
- (b) Satu pagar bersisi tiga akan dibina bersebelahan dengan satu bahagian lurus sebatang sungai yang menjadi sisi keempat kepada satu kawasan segi empat tepat. Terdapat 96 meter pagar yang tersedia. Cari luas maksimum kawasan tertutup tersebut dan dimensi kawasan segi empat tepat tersebut.
- [10 markah]

Question 5

Evaluate the following integrals:

- (a) $\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$
- (b) $\int_{-1}^4 |2x - 1| dx$
- (c) $\int \frac{3x}{e^x} dx$ (Use integration by parts)
- (d) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x^2+1}{x^2-1} dx$ (Use partial fractions)

[30 marks]

Soalan 5*Nilaikan kamiran berikut:*

- (a) $\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$
- (b) $\int_{-1}^4 |2x - 1| dx$
- (c) $\int \frac{3x}{e^x} dx$ (Gunakan pengamiran bahagian demi bahagian)
- (d) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x^2+1}{x^2-1} dx$ (Gunakan pecahan separa)

[30 markah]

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SULIT**Question 6**

Sketch the region bounded by $x = y^2 + 2$, $y = x - 4$ and $y = 0$. Then, set up, but do not evaluate, the integral for

- (a) the area of the region,
- (b) the volume of the solid generated when the region is revolved about the x -axis, and
- (c) the volume of the solid generated when the region is revolved about the y -axis.

[20 marks]

Soalan 6

Lakarkan rantau yang dibatasi oleh $x = y^2 + 2$, $y = x - 4$ dan $y = 0$. Seterusnya, nyatakan kamiran, tanpa menilaikannya untuk

- (a) luas rantau,
- (b) isipadu pepejal yang dihasilkan apabila rantau tersebut diputarkan sekitar paksi- x , dan
- (c) isipadu pepejal yang dihasilkan apabila rantau tersebut diputarkan sekitar paksi- y .

[20 markah]

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